M/EEG source analysis

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Key points:

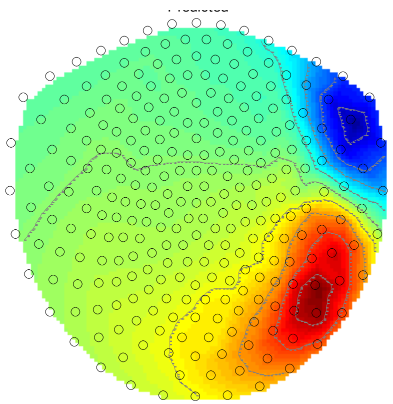
- What is an ill-posed inverse problem?

- Prior knowledge -links to popular algorithms

- Validation of prior knowledge / Model evidence

The forward problem

Prediction



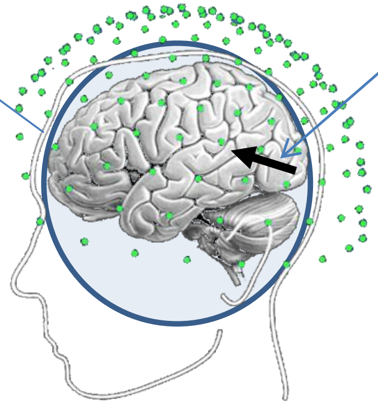
M/EEG sensors

Lead field (L) is the sensitivity of the M/EEG system to a dipolar source at a particular location

Lead fields (L)

Head Model

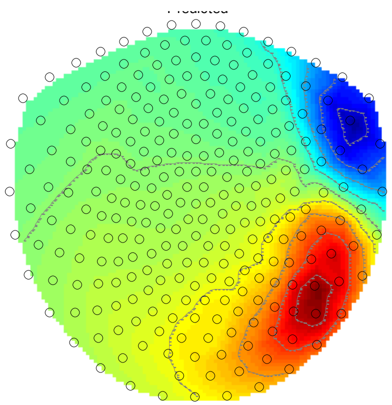
Dipolar source



Model describes conductivity & geometry

Analogy
2+3= ?

The Inverse problem



M/EEG
sensors

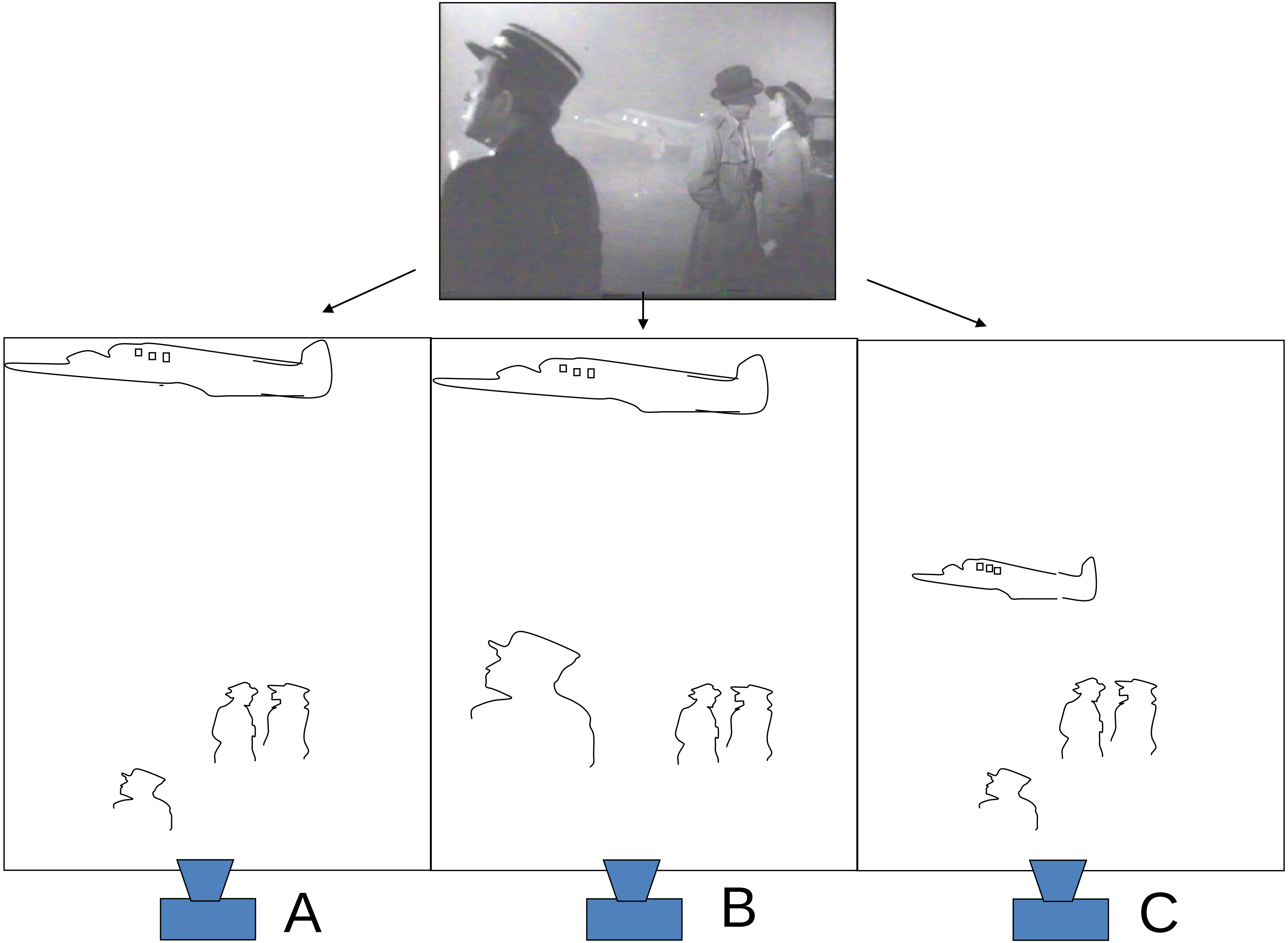
Measurement

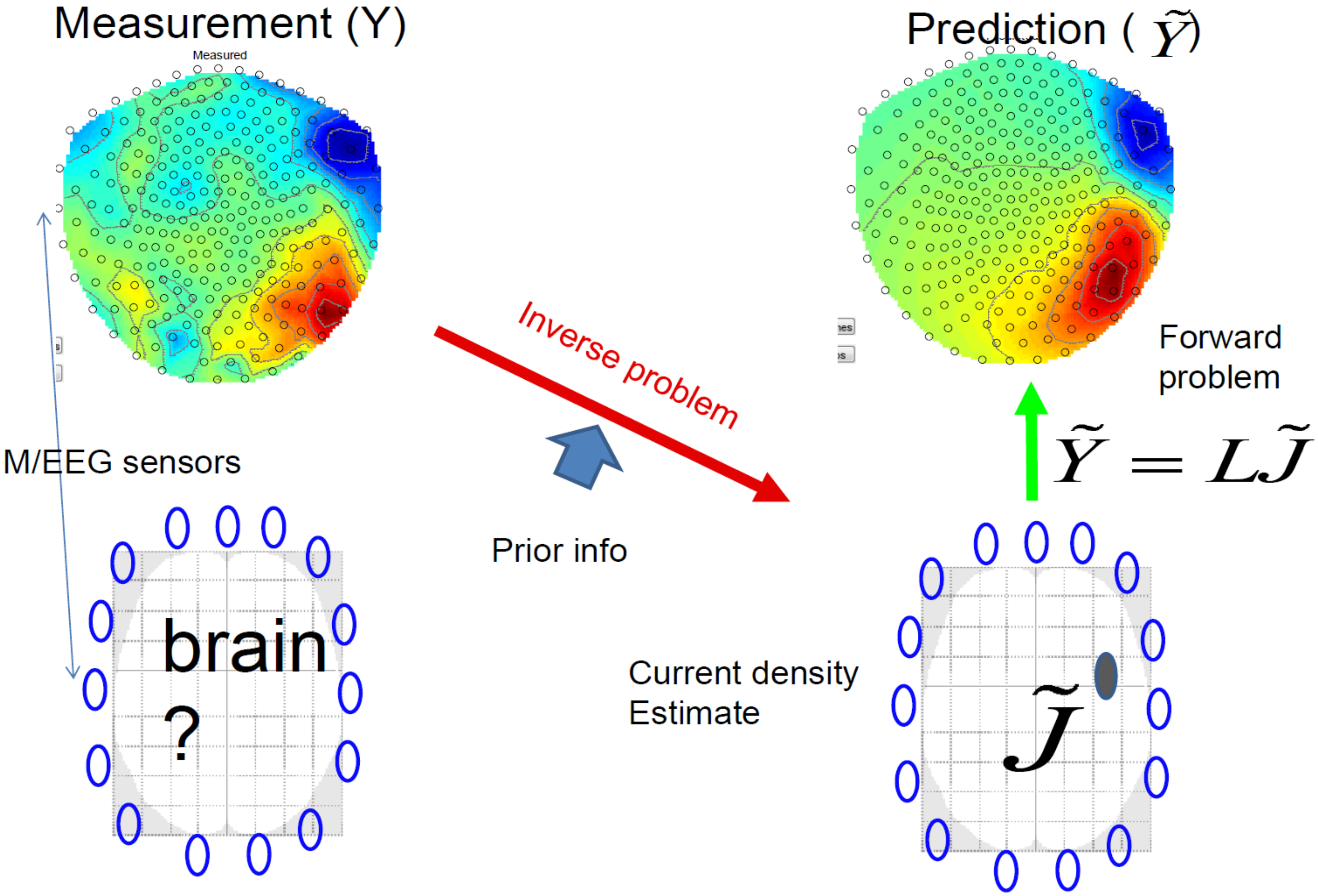
Which brain sources gave rise to these measured data ?

Inference

Analogy
5= ? +?

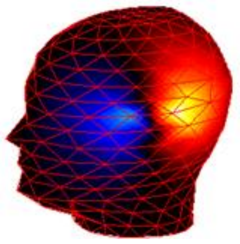
Inverse problems aren’t difficult


A
B
C

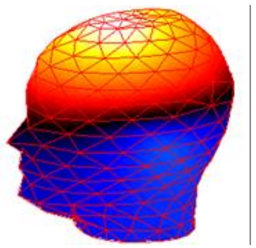

Measurement (Y)
Measured
Prediction ($\tilde{Y}$)
Inverse problem
Forward problem
$\tilde{Y} = L\tilde{J}$
M/EEG sensors
Prior info
brain ?
Current density Estimate
$\tilde{J}$

The forward problem

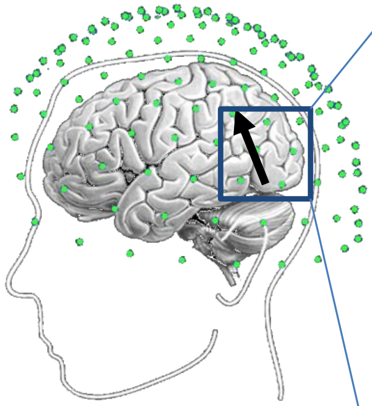
MEG



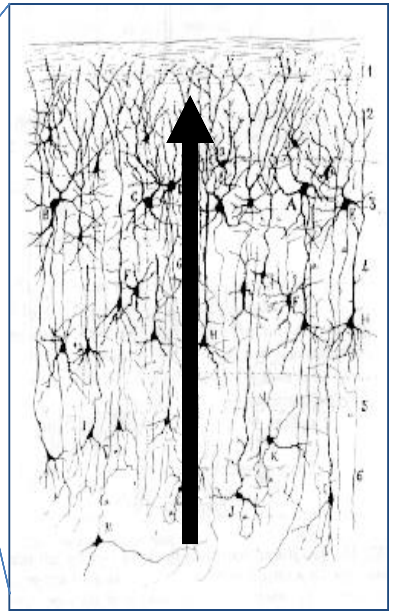
EEG



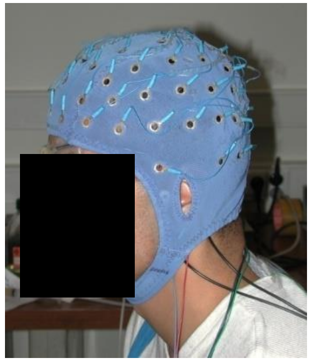
Lead fields



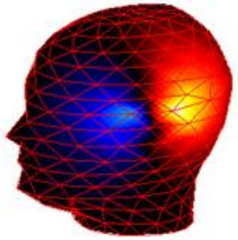
Head tissues
(conductivity & geometry)



Neurons

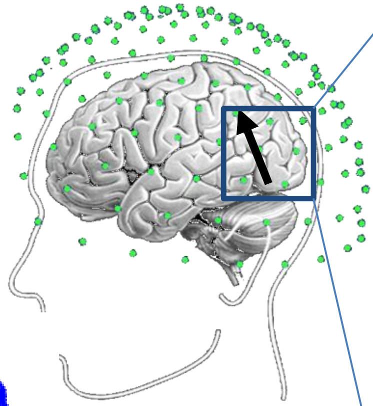


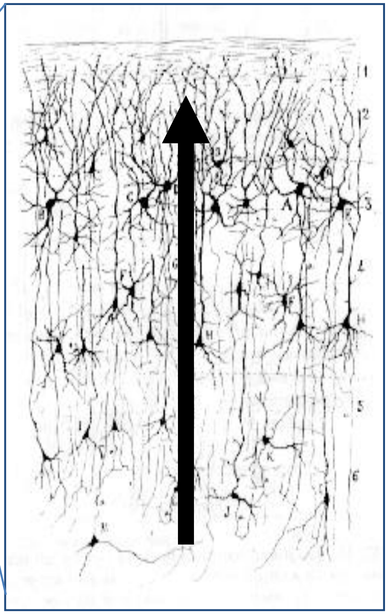
The forward problem



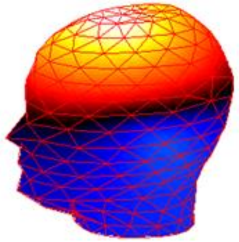
MEG

Lead fields

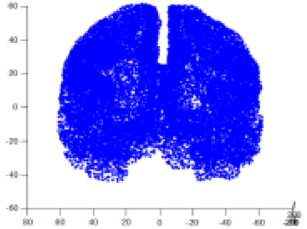


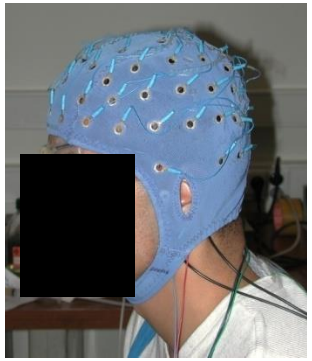


Dipolar sources



EEG



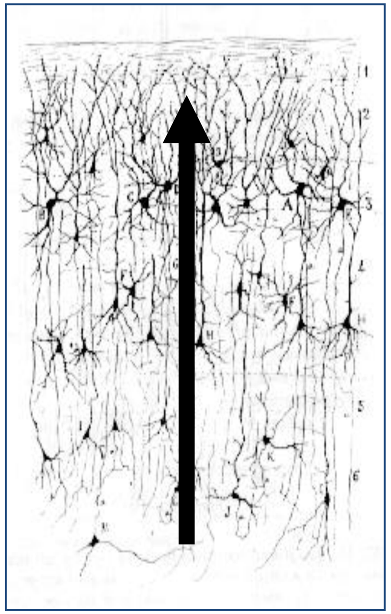


$$Y = LJ + \epsilon$$

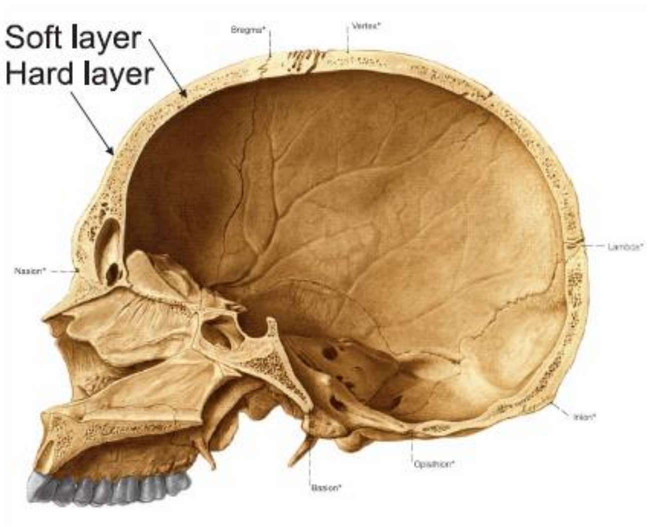
Noise

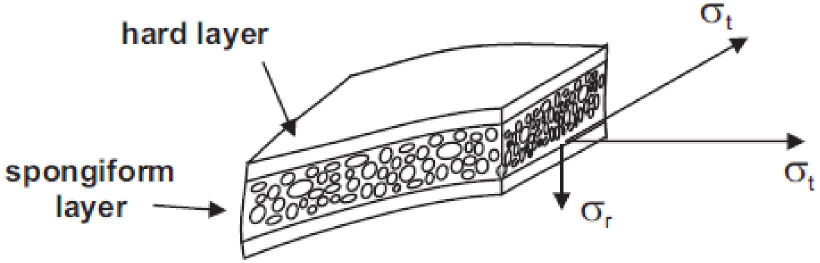
Head model

Head model



Neurons



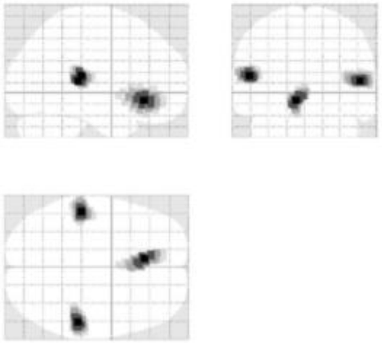


MEG/EEG brain imaging

With the acquired data we may recover the neural activity

$$J = L^{-1} Y$$

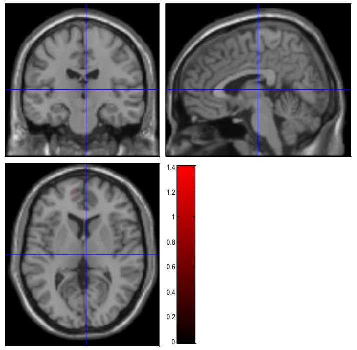
dipoles



samples

=

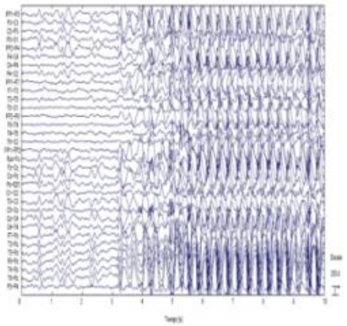
dipoles



sensors

×

sensors



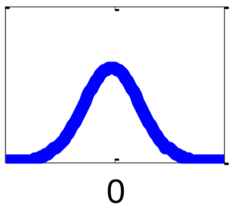
samples

But the problem is ill-posed: # dipoles >> # sensors

NON INVERTIBLE!!! → Infinite solutions!!!

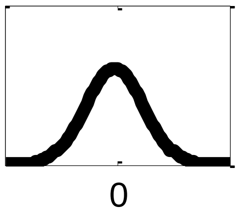
Bayesian formulation

Q



We must include prior information:

$$Y = LJ + \epsilon$$



Σ_ϵ

then we can use the Bayes' theorem:

Forward problem (Adjusted with the data)

Prior (Assumed)

$$p(J|Y) = \frac{p(Y|J)p(J)}{p(Y)}$$

Evidence (constant)

and solving for Gaussian assumptions:

$$\widehat{J} = E[p(J|Y)] \rightarrow \boxed{\widehat{J} = QL^T(\Sigma_\epsilon + LQL^T)^{-1}Y}$$

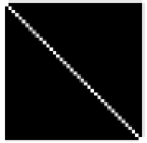
Grech et al., 2008

Prior covariance matrices

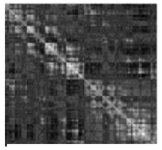
PRIOR NOISE COVARIANCE

Independent sensor noise

$$\Sigma_\epsilon = h_0 I$$

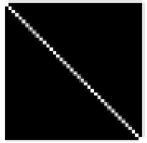


Empty room activity



PRIOR COVARIANCE OF SOURCE SPACE ACTIVITY

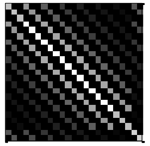
Minimum norm $Q = I$ →



Non informative

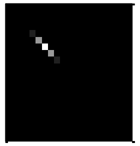
$$\widehat{J} = L(\Sigma_\epsilon + LL^T)^{-1} Y$$

LORETA-like: $Q = e^{\sigma G_L}$ →



Smoothed

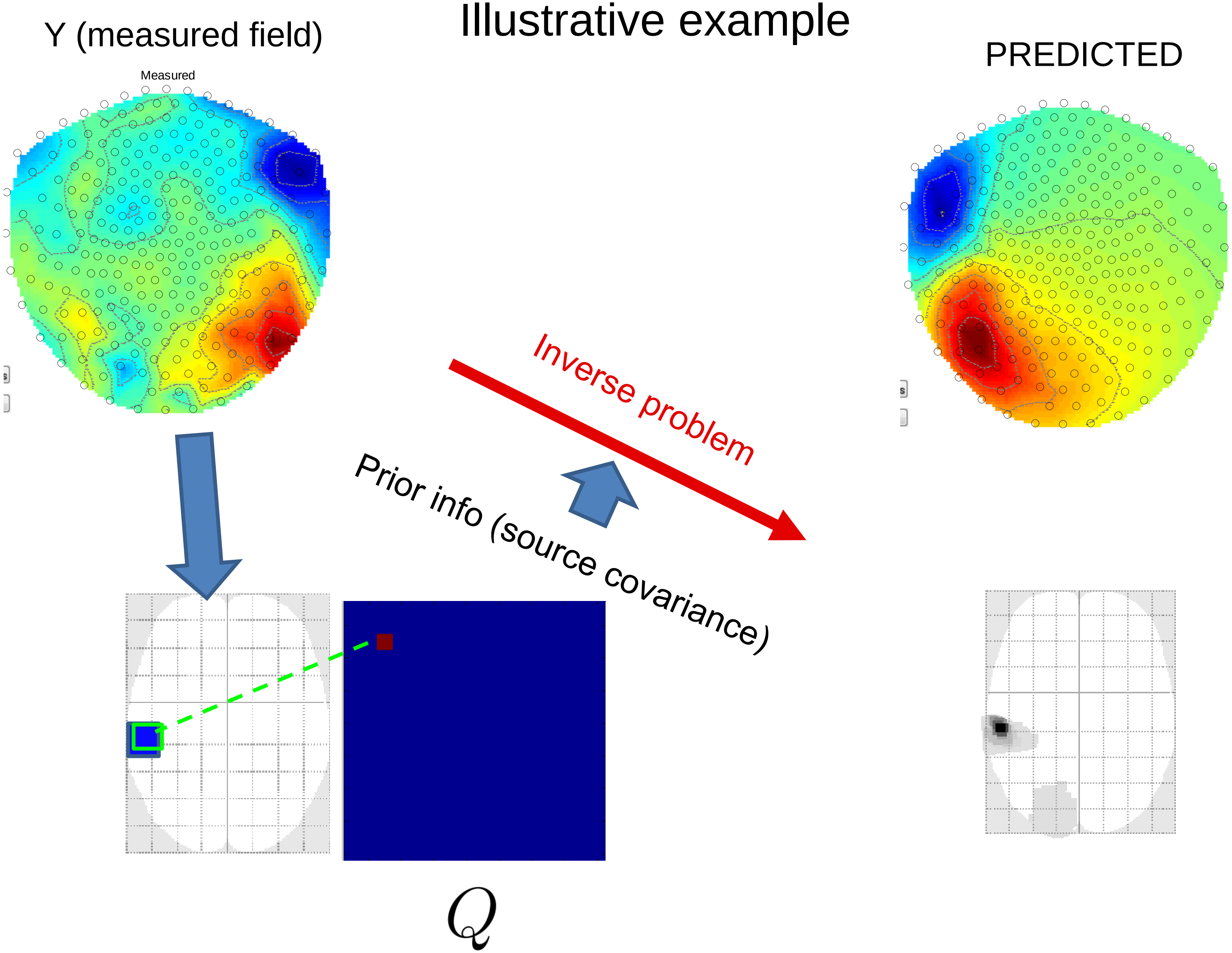
Beamformers: $Q = (L^T (YY^T)^{-1} L)^{-1}$ →

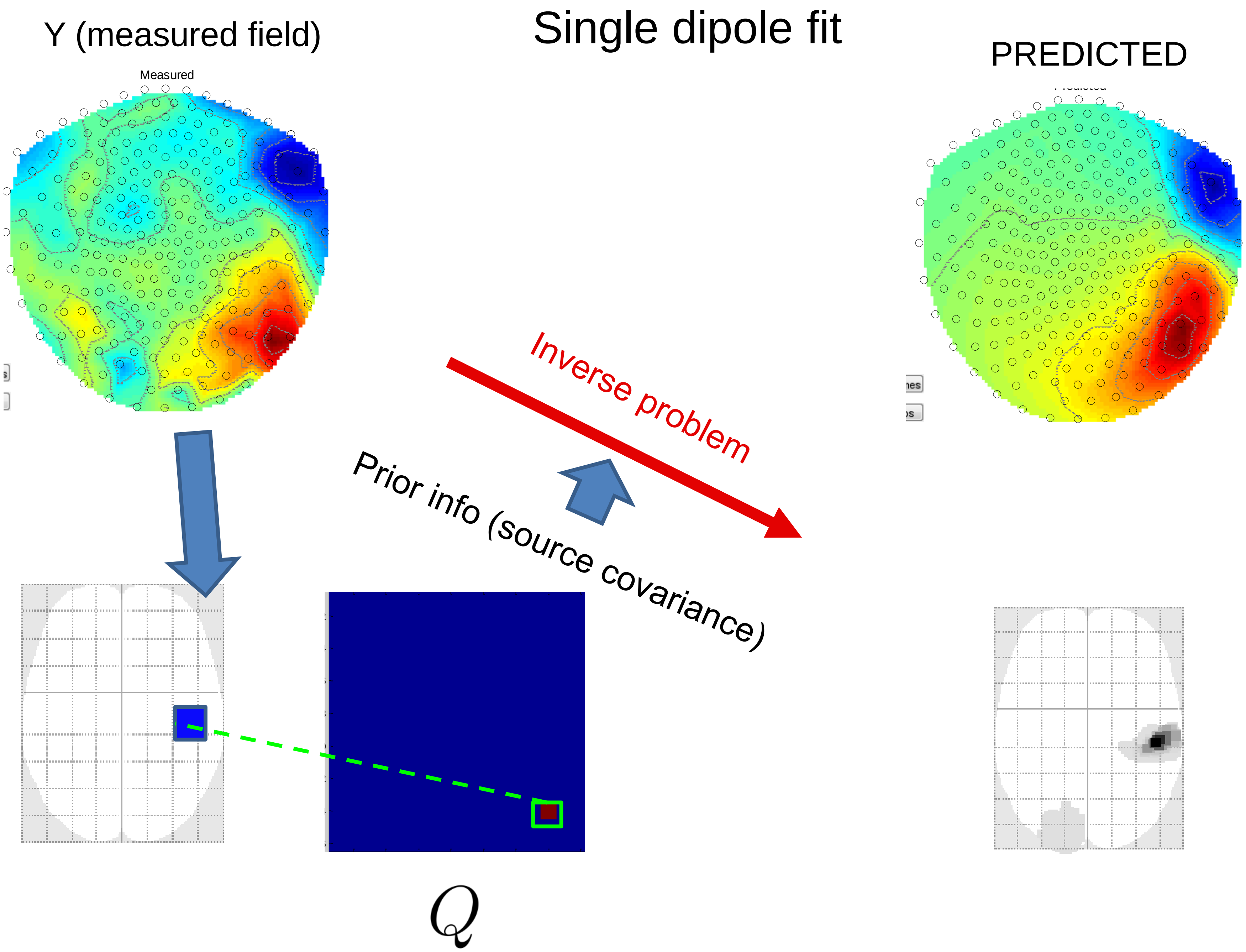


Data based

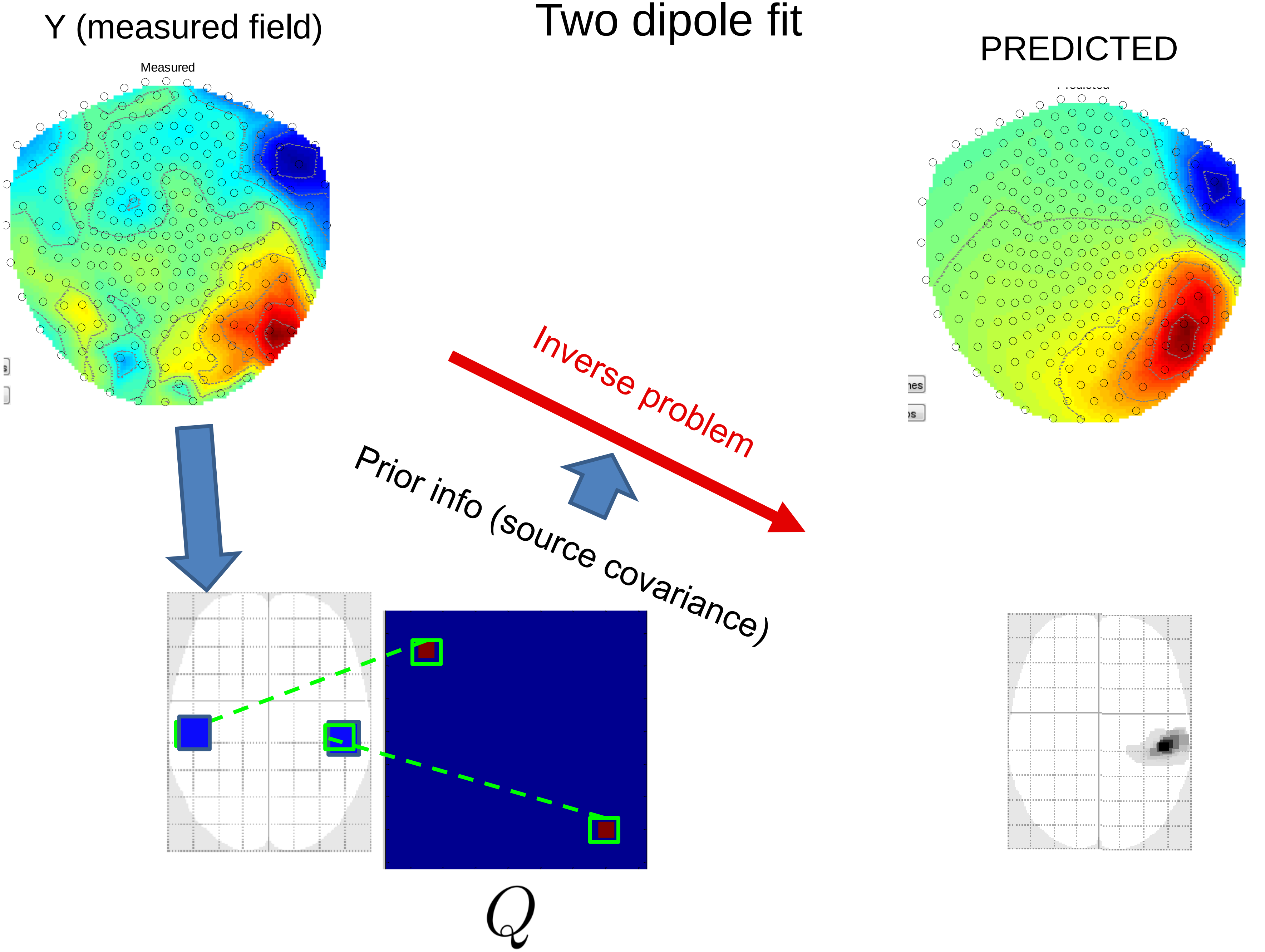
How do they work?

Illustrative example

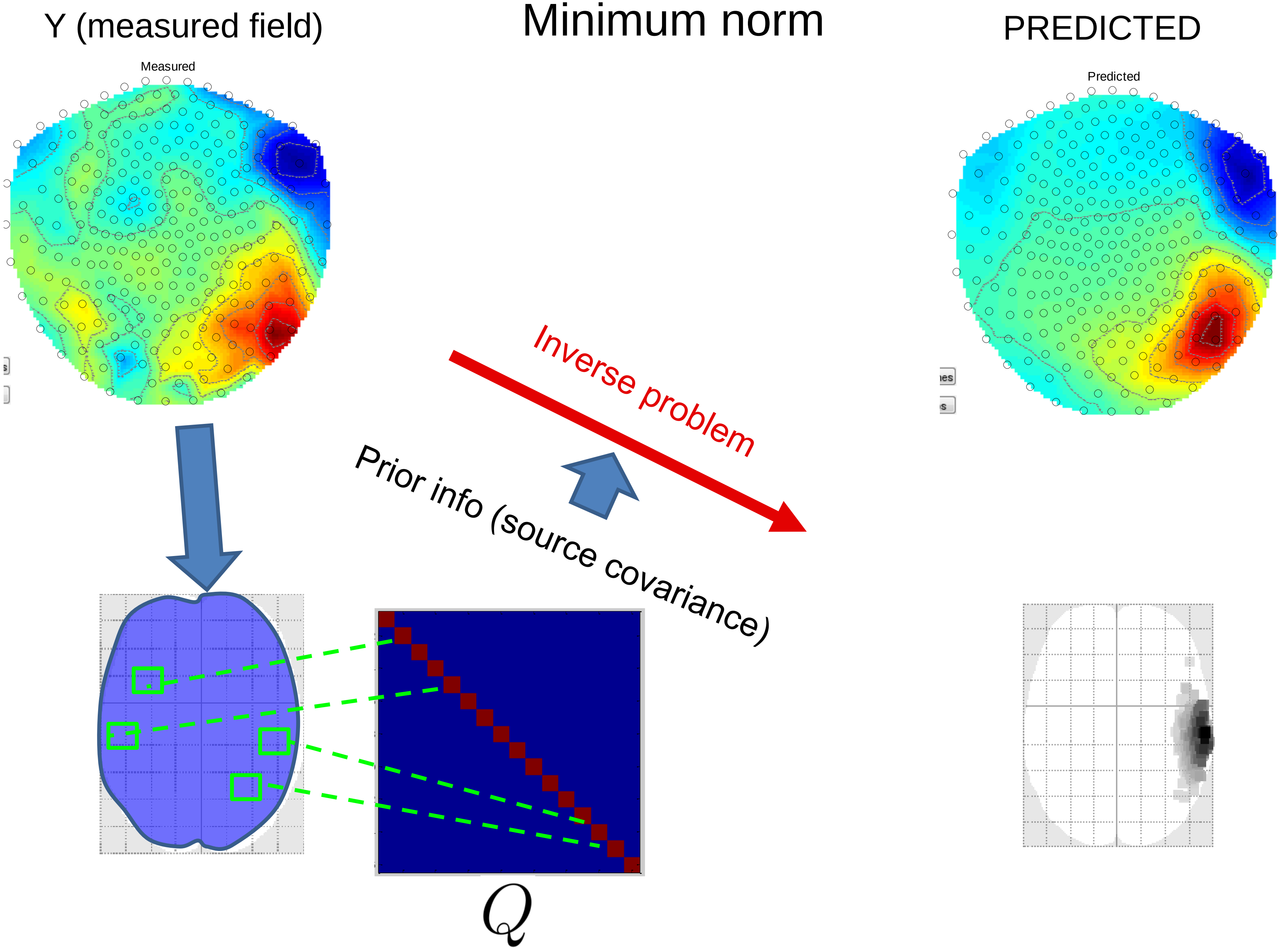



Single dipole fit
Y (measured field)
Measured
PREDICTED
Inverse problem
Prior info (source covariance)
Q

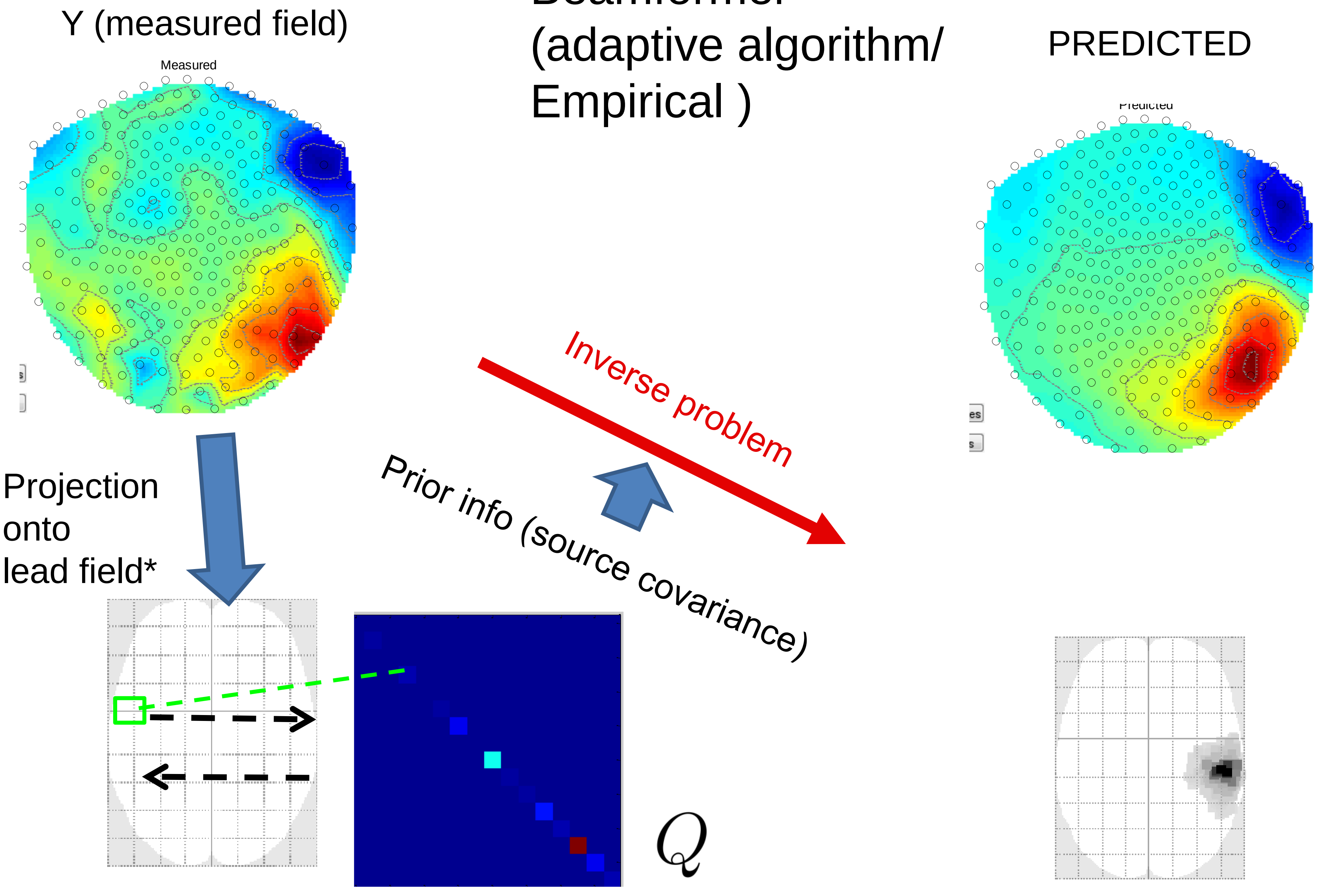
Two dipole fit



Minimum norm



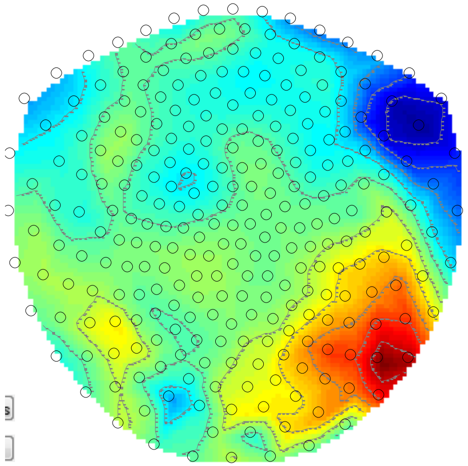
Beamformer (adaptive algorithm/ Empirical)



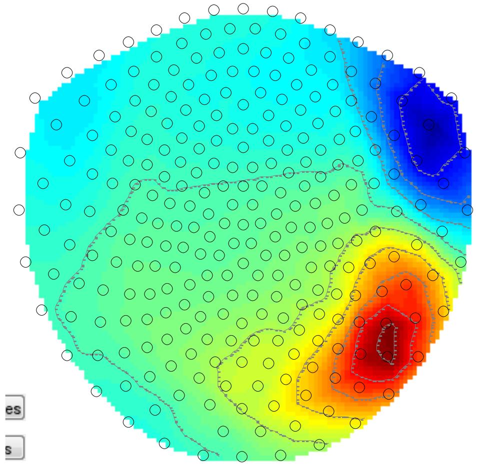
*Assuming no correlated sources

fMRI biased dSPM
(Dale et al. 2000)

Y (measured field)

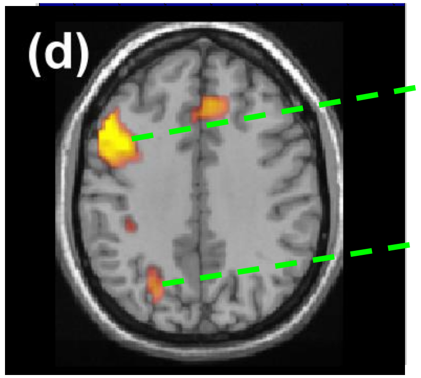


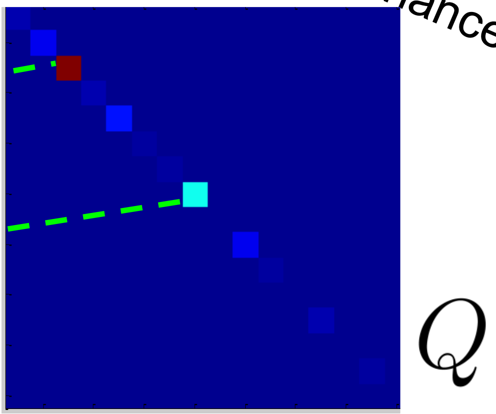
PREDICTED



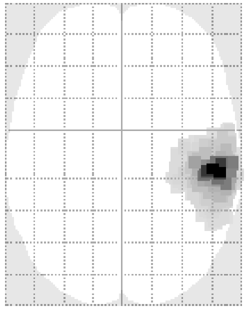
Inverse problem

Prior info (source covariance)



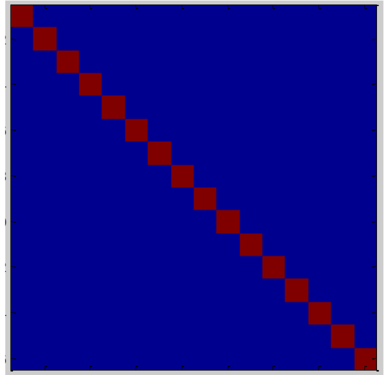


fMRI data

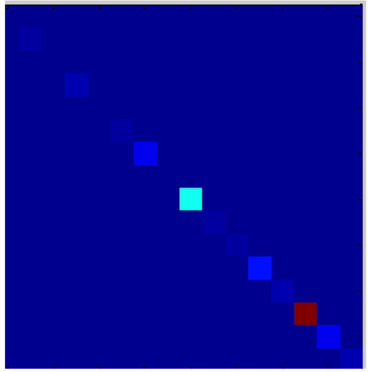


Maybe…

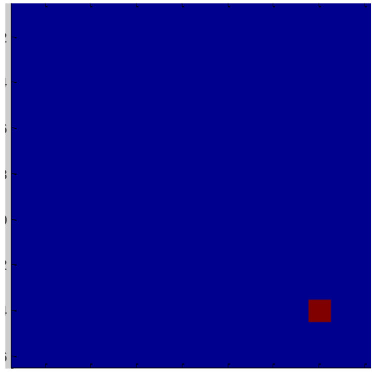
Summary: Some popular priors



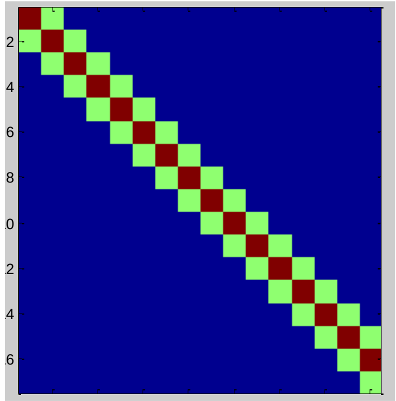
Minimum norm



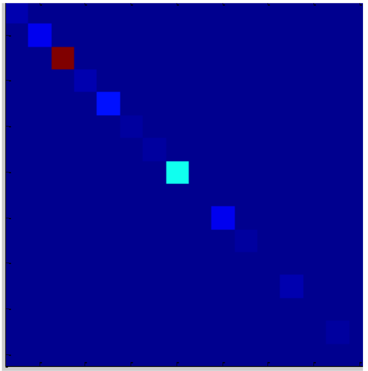
SAM,DICs
Beamformer



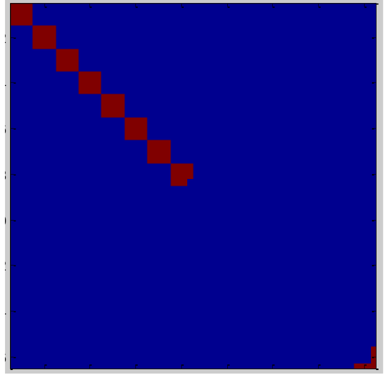
Dipole fit
(Non-linear)



LORETA

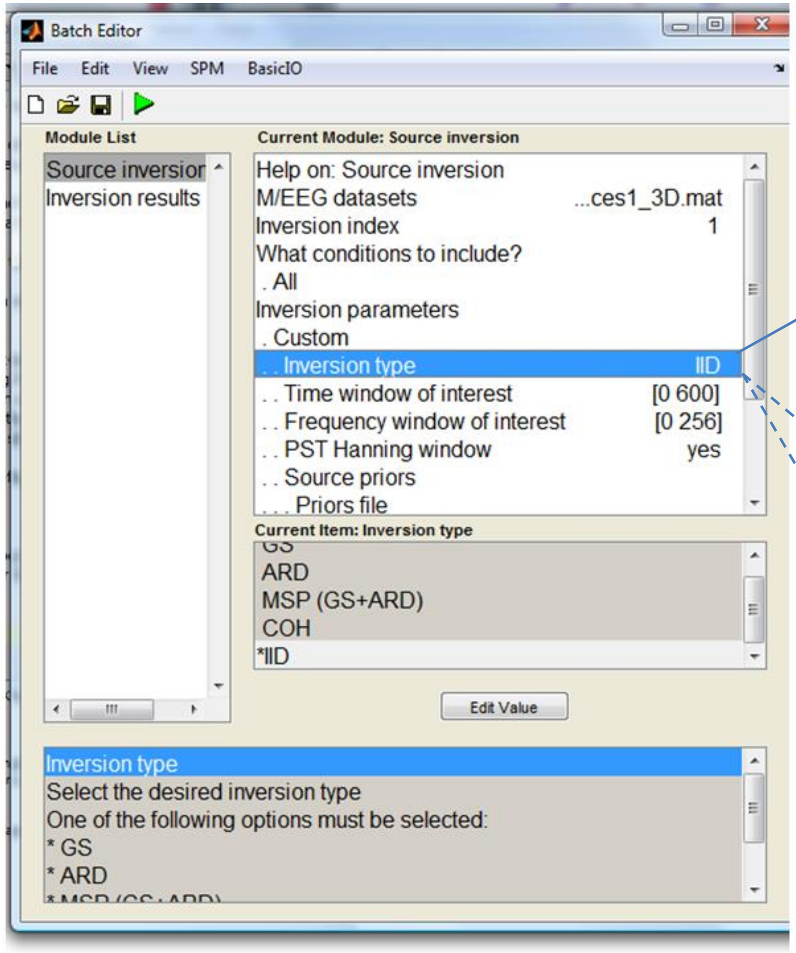


fMRI biased
dSPM



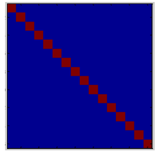
?

SPM12

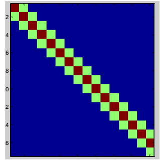


Minimum Norm (IID

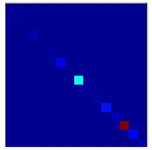
- independent and identically distributed)



LORETA-like (COH- coherent)



Empirical Bayes Beamformer (EBB)



Multiple Sparse Priors

(MSP/ Greedy Search (GS)

Automatic relevance determination (ARD))

Software

- ***SPM12:* *http://www.fil.ion.ucl.ac.uk/spm/software/spm12/***

- ***DAiSS-*** SPM12 toolbox for Data Analysis in Source Space (beamforming, minimum norm and related methods), developed by Vladimir Litvak:

  https://github.com/spm/DAiSS

- **Fieldtrip**: http://fieldtrip.fcdonders.nl/
- **Brainstorm**: http://neuroimage.usc.edu/brainstorm/
- **MNE**: http://martinos.org/mne/stable/index.html

Summary

- MEG/EEG inverse problem requires prior information in the form of a source covariance matrix.

- Different inversion algorithms- SAM, DICS, LORETA, Minimum Norm, dSPM… just have different prior source covariance structure.

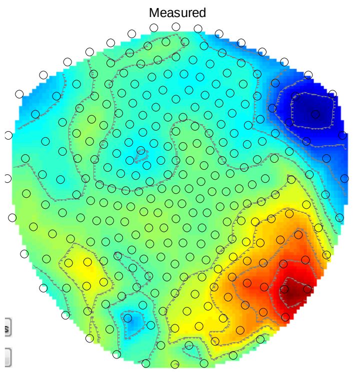
- Historically- different MEG groups have tended to use different algorithms/acronyms.

See
Mosher et al. 2003, Friston et al. 2008, Wipf and Nagarajan 2009, Lopez et al. 2014

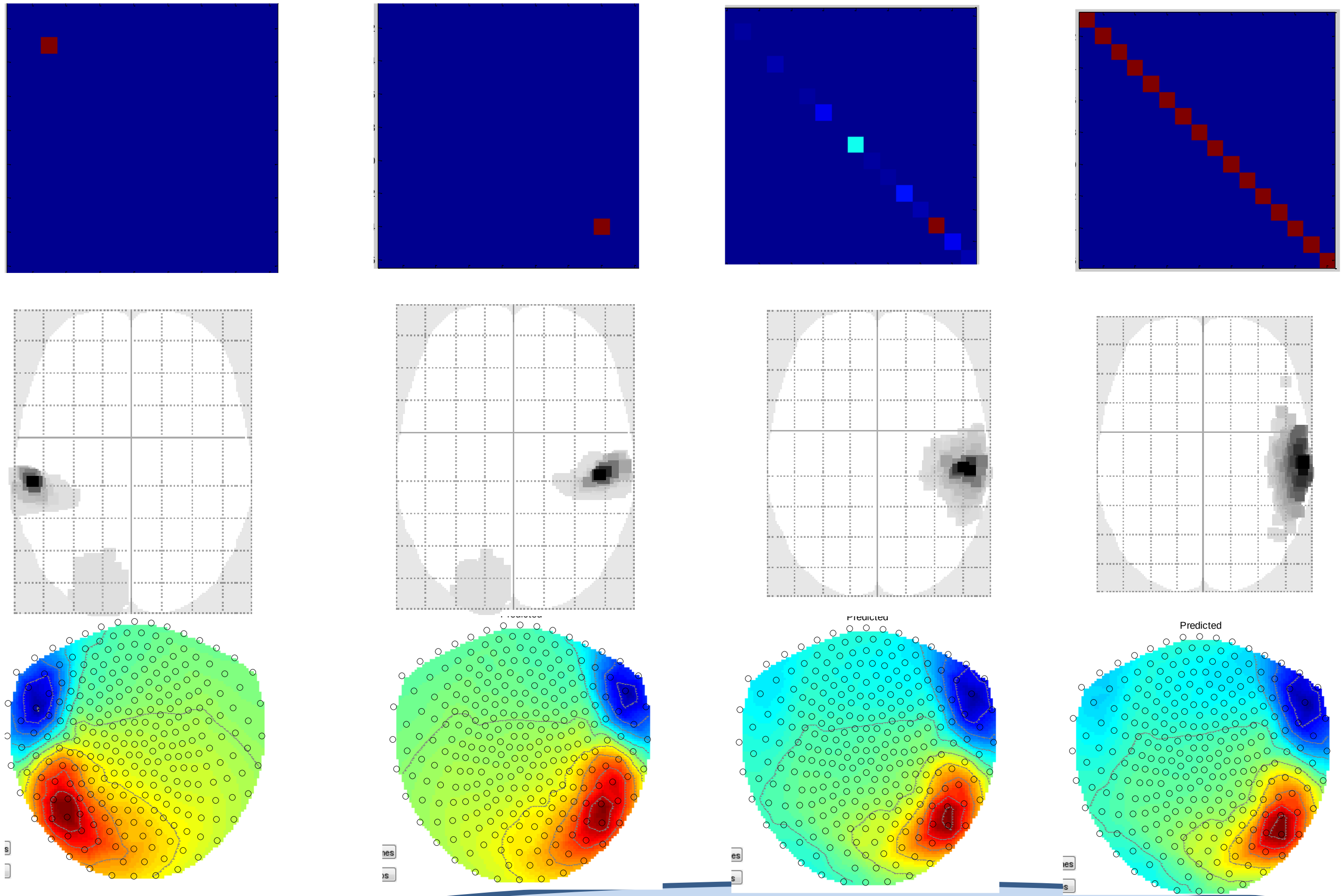
How can I choose?

How do we chose between priors ?

Y (measured field)



	Incorrect prior	Ground truth	Beamformer	Minimum norm
Prior				
Estimated Current flow				
Predicted data				
Variance explained	11 %	96%	97%	98%



Penny, 2012

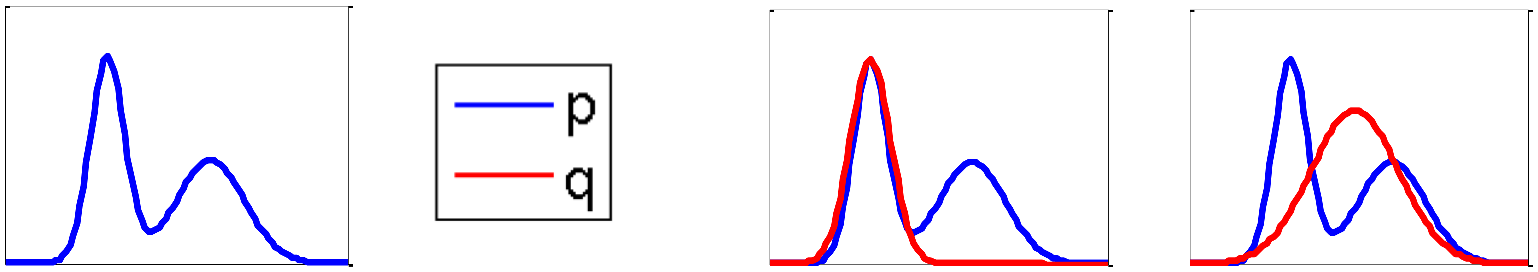
Negative variational free energy (1)

$$\boxed{\log p(\widehat{Y}) = F + KL[q(h)||p(h|Y)]}$$

the divergence will be zero if the approximated distribution is equal to the posterior one:

$$q(h) = p(h|Y) \Longrightarrow F = \log p(Y)$$

$$q_0(h) = \mathcal{N}(h; \nu, \Pi^{-1}) \Longrightarrow q(h) = \mathcal{N}(h; \widehat{h}, \Sigma_h)$$



$p(h|Y)$ Wrong **BETTER!**

This is from my PhD. thesis:

Define the log evidence as:

$$\log p(\boldsymbol{Y}) = \int q(\boldsymbol{h}) \log p(\boldsymbol{Y}) dh$$

Applying the definition of Eq. (3-6), log evidence can be extended to:

$$\begin{aligned}\log p(\boldsymbol{Y}) &= \int q(\boldsymbol{h}) \log \frac{p(\boldsymbol{Y}, \boldsymbol{h})}{p(\boldsymbol{h}|\boldsymbol{Y})} dh = \int q(\boldsymbol{h}) \log \frac{p(\boldsymbol{Y}, \boldsymbol{h}) q(\boldsymbol{h})}{q(\boldsymbol{h}) p(\boldsymbol{h}|\boldsymbol{Y})} dh \\ &= \int q(\boldsymbol{h}) \log \frac{p(\boldsymbol{Y}, \boldsymbol{h})}{q(\boldsymbol{h})} dh + \int q(\boldsymbol{h}) \log \frac{q(\boldsymbol{h})}{p(\boldsymbol{h}|\boldsymbol{Y})} dh \\ &\boxed{= F + \mathrm{KL}[q(\boldsymbol{h}) || p(\boldsymbol{h}|\boldsymbol{Y})]}\end{aligned}$$

Negative variational free energy (2)

The free energy can be expressed as:

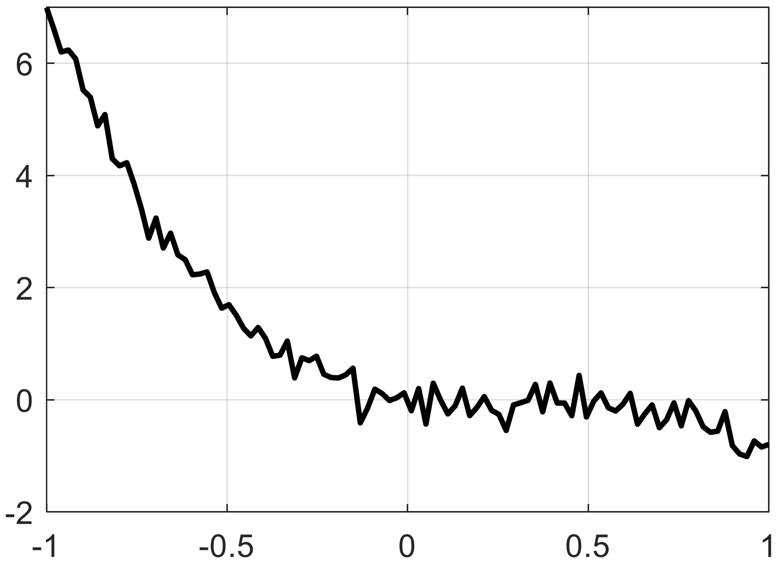
$$\boxed{\begin{aligned} F = &-\frac{N_t}{2}\mathrm{tr}(C_Y\Sigma_Y^{-1}) - \frac{N_t}{2}\log|\Sigma_Y| - \frac{N_cN_t}{2}\log(2\pi) \\ &-\frac{1}{2}\mathrm{tr}\left((\widehat{h}-\nu)^T\Pi(\widehat{h}-\nu)\right) + \frac{1}{2}\log|\Pi\Sigma_h| \end{aligned}}$$

Reducing constant terms and assuming zero mean priors:

$$\begin{aligned} F = &-\begin{bmatrix}\text{Model}\\ \text{error}\end{bmatrix} - \begin{bmatrix}\text{Size of model}\\ \text{covariance}\end{bmatrix} - \begin{bmatrix}\text{Num of data}\\ \text{samples}\end{bmatrix} \rightarrow \textbf{Accuracy} \\ &-\begin{bmatrix}\text{Error in}\\ \text{hyperparameters}\end{bmatrix} + \begin{bmatrix}\text{Error in covariance}\\ \text{of hyperparameters}\end{bmatrix} \rightarrow \textbf{Complexity} \end{aligned}$$

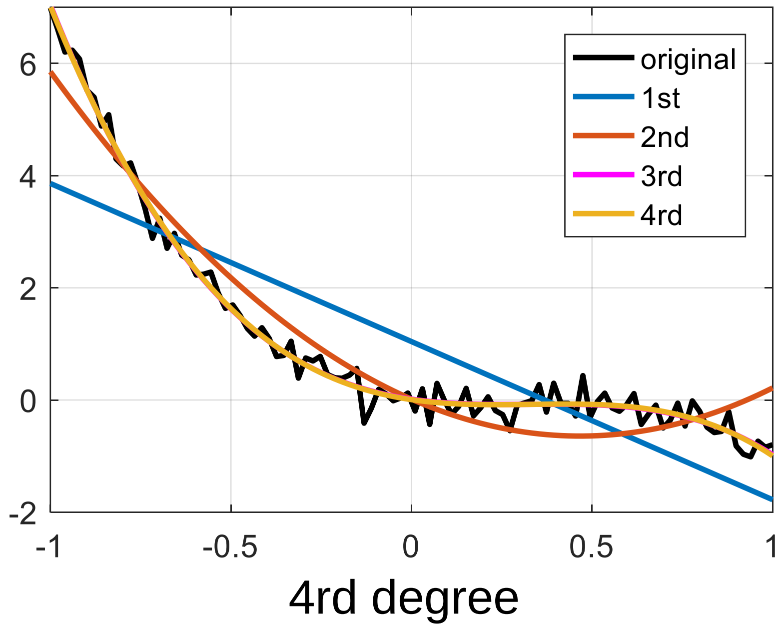
Trade-off between accuracy and complexity

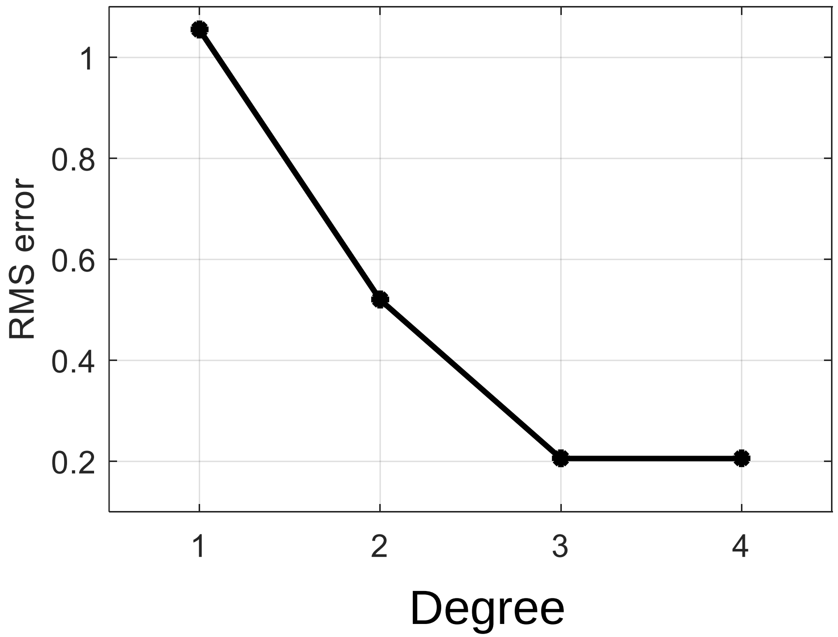
Approach	**Complexity term**
AIC (Akaike, 1974)	N_q
BIC (Schwarz, 1978)	$\frac{N_q}{2} \log N_t$
Linear function (Wipf and Nagarajan, 2009)	h
free energy (Friston et al., 2008)	$\frac{1}{2}\mathrm{tr}\left((h-\nu)^T \Pi (h-\nu)\right) - \frac{1}{2}\log\lvert\Pi\Sigma_h\rvert$



Function to estimate

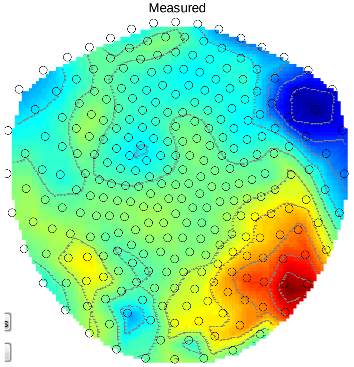
Trade-off between accuracy and complexity

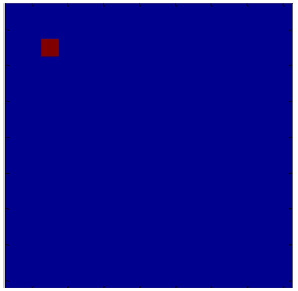
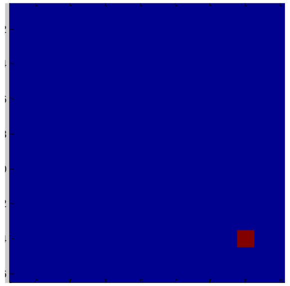
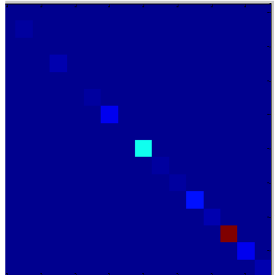
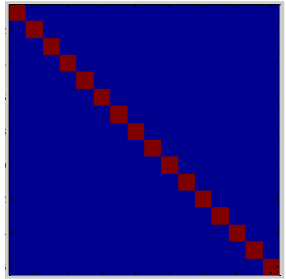
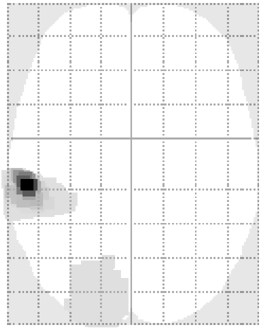
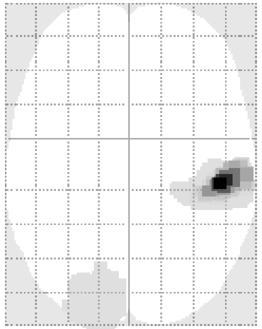
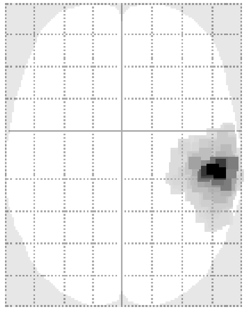
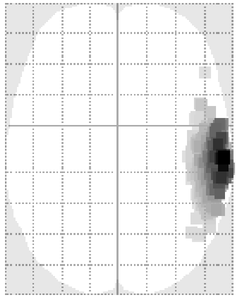
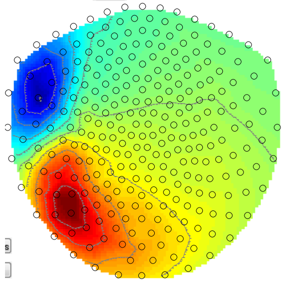
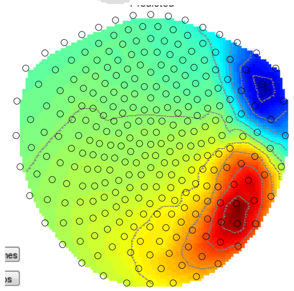
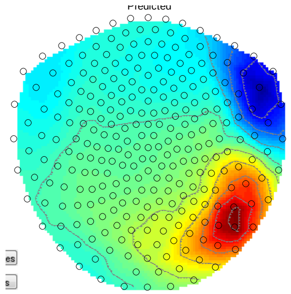
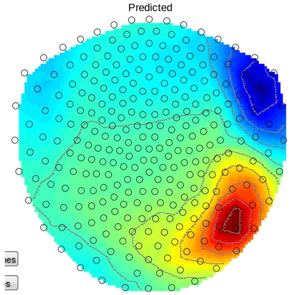




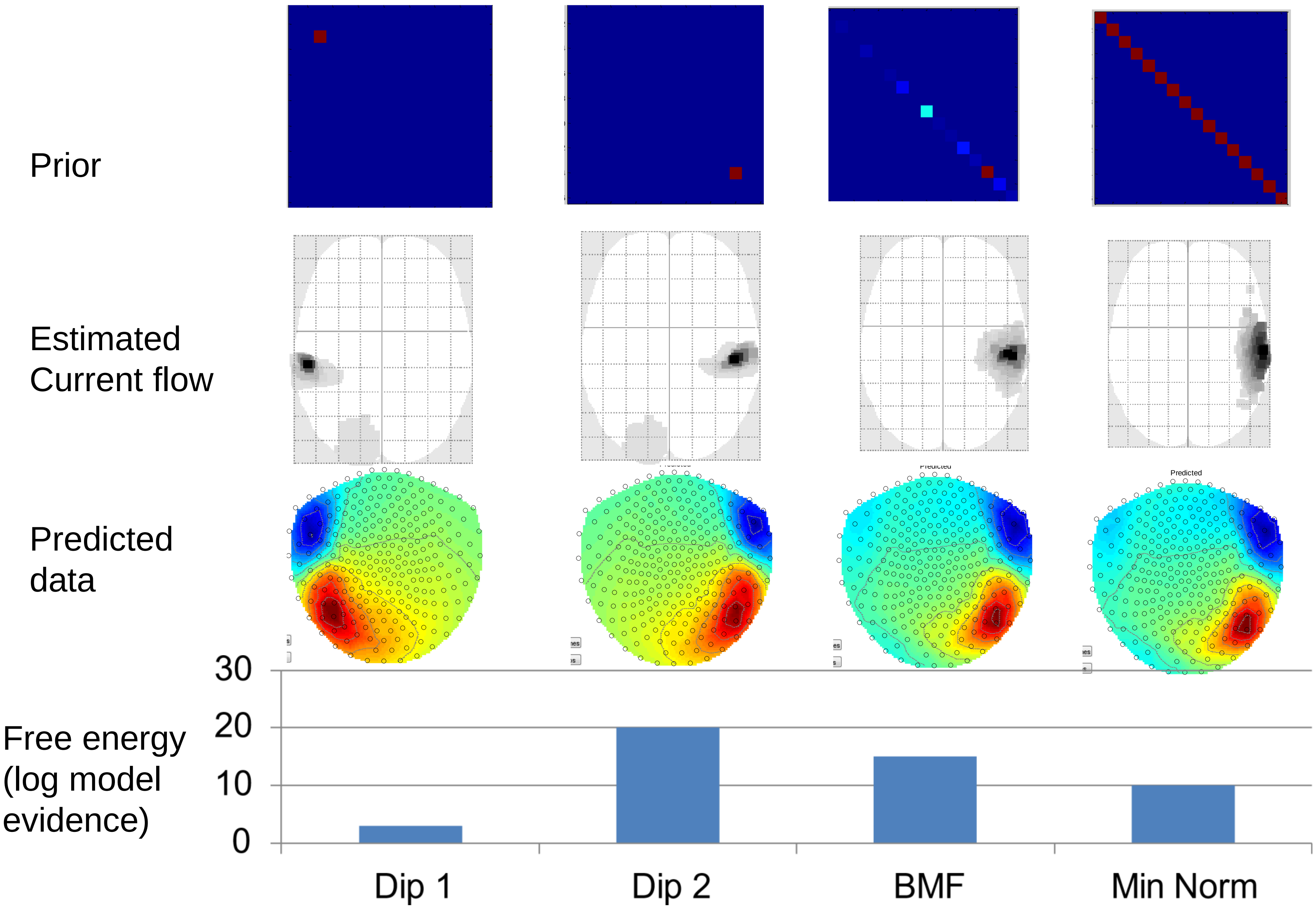
How do we chose between priors ?

Y (measured field)



	Incorrect prior	Ground truth	Beamformer	Minimum norm
Prior				
Estimated Current flow				
Predicted data				
Variance explained	11 %	96%	97%	98%

How do we chose between priors ?



Multiple Sparse Priors (MSP)

Friston et al., 2008

Multiple sparse priors (1)

All prior information can be included as the linear combination of a set of covariance components

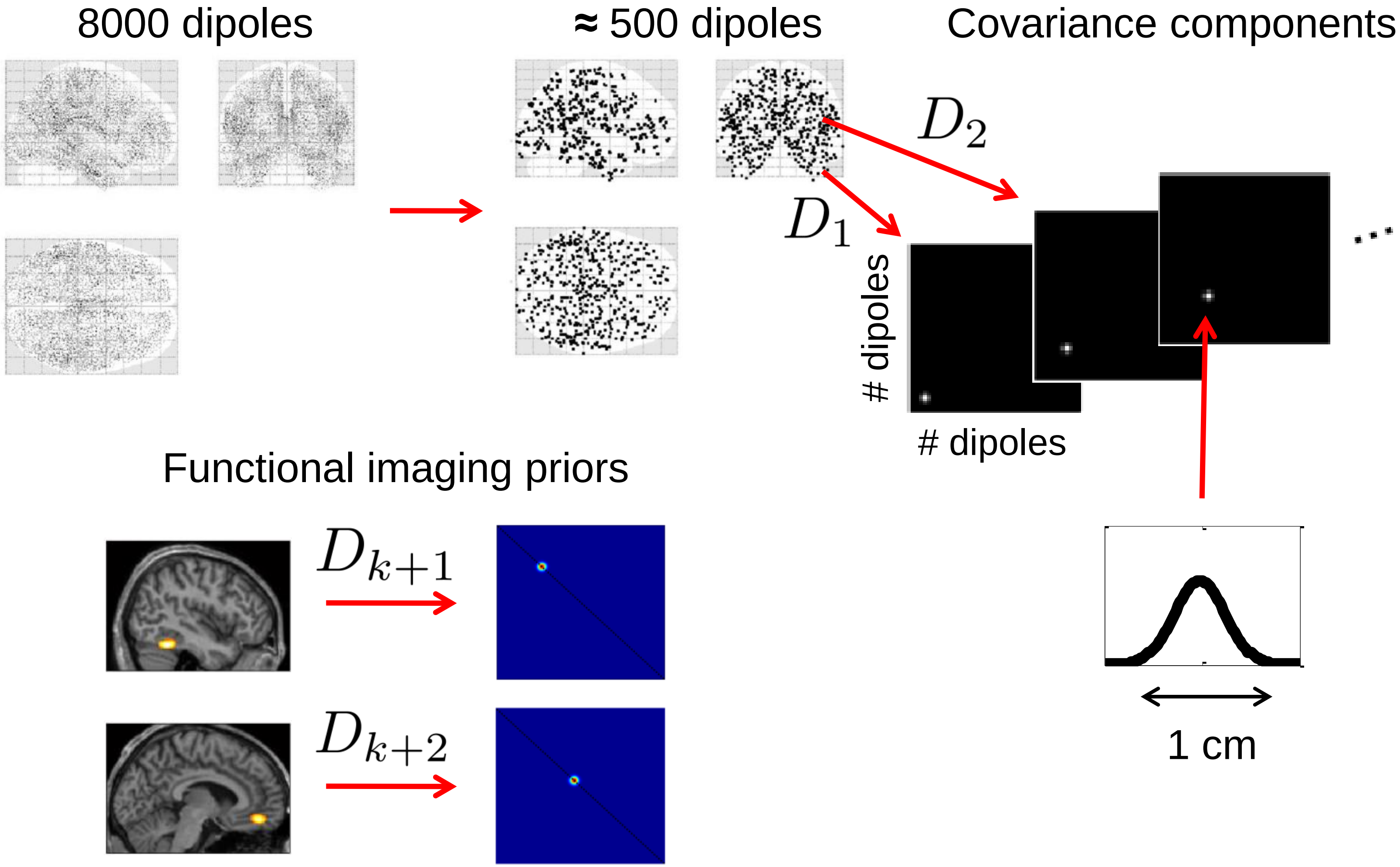
$$\widehat{J} = QL^T(\Sigma_\epsilon + LQL^T)^{-1}Y$$

$$\boxed{Q = \sum_{i=1}^{N_q} h_i D_i}$$

$$D = \{D_1, \ldots, D_{N_q}\}$$

$$h = \{h_1, \ldots, h_{N_q}\}$$

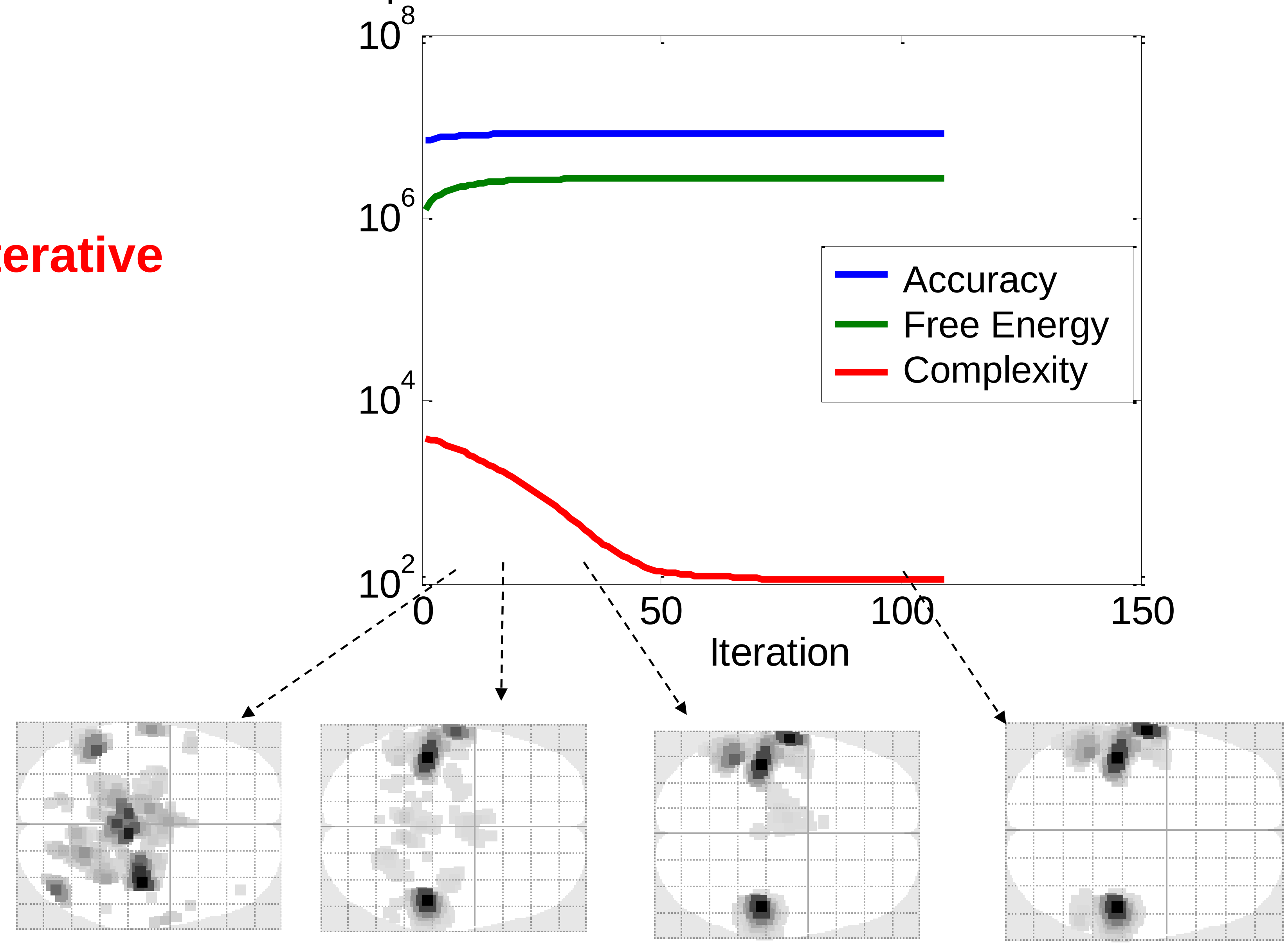
Multiple sparse priors (2)



Multiple Sparse priors

So now construct the priors to maximise model evidence

Iterative



Conclusion

- M/EEG inverse problem can be solved... If you have some prior knowledge.

- All prior knowledge encapsulated in a source covariance matrix Q.

- Can test among priors (or develop new priors) within a Bayesian framework.

References

NeuroImage 84 (2014) 476–487

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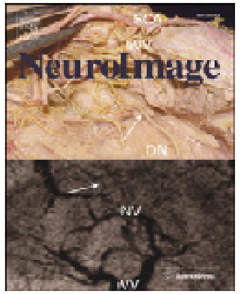

Technical Note

Algorithmic procedures for Bayesian MEG/EEG source reconstruction in SPM☆

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ABSTRACT

The MEG/EEG inverse problem is ill-posed, giving different source reconstructions depending on the initial as-

Thank you

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And all SPM developers